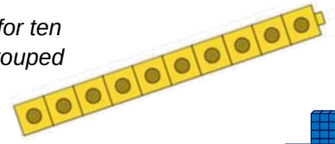
Year 1
Key Methods and Representations

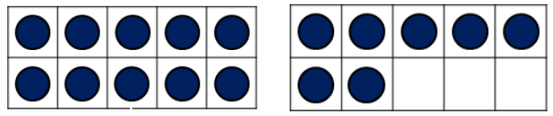
Representations of number

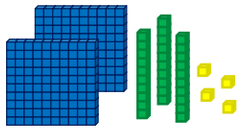
Pupils use counters, cubes and other discrete objects to represent number. Cubes are used to support the process of **regrouping** – one ten is equal to ten ones. A ten frame supports this alongside number bonds for 10. Both are used to represent teen numbers.

One ten is regrouped for ten ones. Ten ones is regrouped for one ten.



17 is one ten and seven ones.

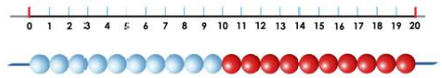




Pupils have also encountered Dienes equipment to represent larger integers to 100. Counting in tens to identify these numbers has also been developed.

Number lines

Number lines are used to represent and compare numbers and can be used alongside a bead string. They demonstrate the continuous nature of the number system. Pupils have ordered numbers on a number line.



Equations

The phrase **'is equal to'** is used consistently to refer to the = symbol. What is on one side of the symbol is equal to what is on the other side. Equations should be presented with symbols in different positions.

$7 = 3 + 4$

$\square = 10 - 3$

Number bond knowledge

Pupils should be increasingly fluent in number bond recall for all numbers to 20 and use representations to support this.

$17 = 12 + 5$
$17 = 11 + 6$
$17 = 10 + 7$

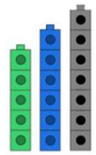
Deriving facts

Pupils use known facts such as number bonds and understanding of place value and magnitude to derive further facts. Commutativity for addition is also used.

If I know $3 + 4 = 7$ *then I know* $13 + 4 = 17$
If I know $3 + 4 = 7$ *then I know* $4 + 3 = 7$

Comparing numbers

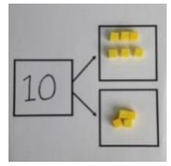
Pupils experience a range of language to compare numbers.



Five is less than seven. Five ones is fewer than seven ones. Seven is greater than five. Six is between five and seven. It is after five and before seven.

Part-whole language and representations

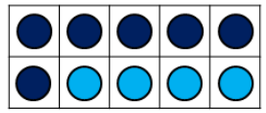
A part-whole model is used to represent the relationship between numbers and will have been used for addition and subtraction. The model is made of a **whole** and two or more **parts**.



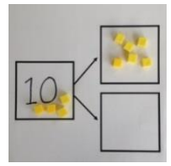
The whole is ten. One part is six and one part is four. Six plus four is equal to ten.

whole = part + part
$10 = 6 + 4$

Different representations have been used to deepen part-whole relationships.



By moving the manipulatives the model represents subtraction.



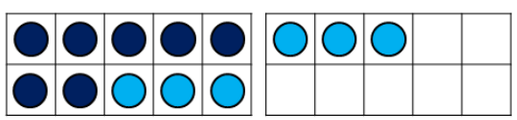
The whole is ten. I subtract one part of six. The missing part is four. Ten subtract six is equal to four.

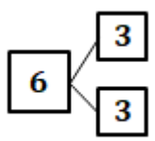
whole – part = part
$10 - 6 = 4$

The 'make 10' strategy

Pupils apply number bonds to 10 to calculate how many more/less to the next multiple of ten. They partition the part into two parts to calculate mentally. Using concrete or pictorial representations can scaffold thinking.

$7 + 6 = ?$ *I know seven and three make 10 so I can partition six into three and three.*



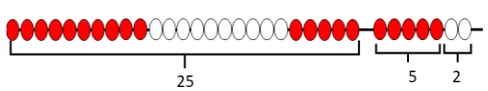


Ten more / ten less

Pupils will explore ten more and ten less than numbers within 50 using manipulatives. They also skip count on and back in tens from different starting points. Mental recall of this can be developed in Maths Meetings.

Finding the difference

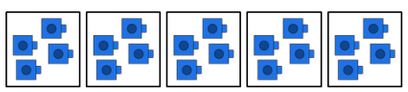
Pupils recognise that in a subtraction calculation where the numbers are close together in value, a count on strategy can be used to find the difference.



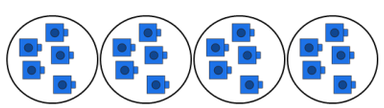
$32 - 25 =?$ *I can count on from 25 to find the difference. Five more is 30, two more is 32. The difference is seven.*

Division by sharing / grouping

Pupils are exposed to the concept of division within 20 through equal grouping and equal sharing. They will explore unequal grouping and sharing. Pupils explore the terms grouping and sharing and be familiar with both.



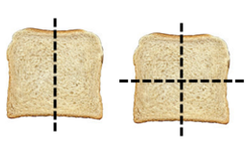
20 shared into five equal groups gives four in each group.



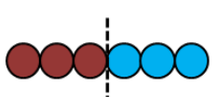
20 grouped into groups of five gives four groups.

Representing fractions

Pupils identify half and quarter of a shape and a quantity within 20 using practical experiences including equal sharing for a quantity. They are also familiar with half turns, linking this to half past on a clock face.

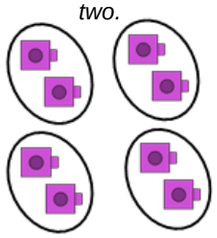


One half is one of two equal parts.
One quarter is one of four equal parts.



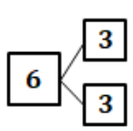
Half of six is three.

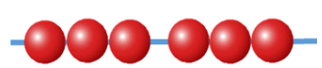
One quarter of eight is two.



Doubling and halving

Pupils have opportunities to represent doubling and halving within 20 using concrete and pictorial representations. This is connected to their understanding of half. Some facts will be recalled and Maths Meetings are an opportunity to consolidate this.





Double three is six. Three plus three is equal to six. Half of six is three. Six take away three is equal to three.

Year 2
Key Methods and Representations

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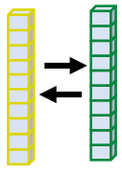
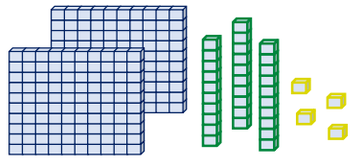
Dienes equipment

An important resource for demonstrating the relative size of place value columns. Supports the process of **regrouping** – one ten is equal to ten ones, one hundred is equal to ten tens and so on. Can also be used to represent addition and subtraction with 2- and 3-digit integers.

One ten is regrouped for ten ones. Ten ones is regrouped for one ten.

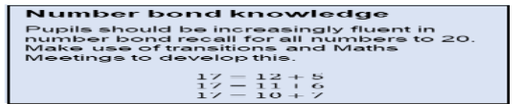


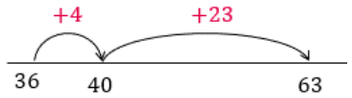


234 is two hundreds, three tens and four ones. I can represent subtracting 20 by removing two ten sticks.

Number lines

Number lines are used to represent and compare numbers and can be used alongside a bead string. They demonstrate the continuous nature of the number system. When calculating, number lines may act as a jotting of the steps of a mental calculation and may begin 'empty' i.e. not have numbered divisions. They are also used when rounding.





Equations

The phrase **'is equal to'** is used consistently to refer to the = symbol. Equations should be presented with symbols and missing numbers in different positions:

$38 = 25 + 13$
$\square = 37 + 44$
$12 \div \square = 4$

Number bond knowledge

Pupils should be increasingly fluent in number bond recall for all numbers to 20. Make use of transitions and Maths Meetings to develop this.

$17 = 12 + 5$
$17 = 11 + 6$
$17 = 10 + 7$

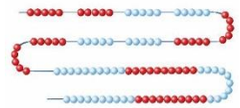
Deriving facts

Pupils use known facts such as number bonds and understanding of place value and magnitude to derive further facts.

If I know $12 + 5 = 17$ *then* $22 + 5 = 27$.
If I know $12 + 5 = 17$ *then* $17 - 12 = 5$
If I know $17 - 12 = 5$ *then* $37 - 12 = 25$

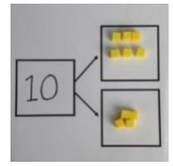
Bead strings

Bead strings help support the ordinality of number. They are repurposed e.g. beads have the value 101-200 for representation when rounding.



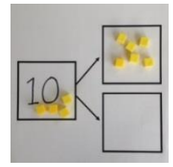
Part-whole language and representations

A part-whole model is used to represent the relationship between numbers in all four operations. The model is made of a **whole** and two or more **parts**.



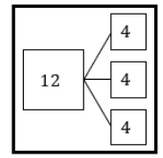
The whole is ten. One part is six and one part is four. Six plus four is equal to ten.

By moving the manipulatives the model represents subtraction.



The whole is ten. I subtract one part of six. The missing part is four. Ten subtract six is equal to four.

Using multiple parts represents multiplication, division and fractions of quantities.

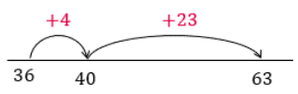


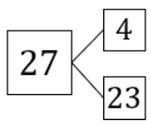
There are three equal parts with a value of four. The whole is 12.
Three multiplied by four is equal to 12.
12 *divided into three equal parts is equal to four.*
One third of 12 *is four.*

The 'make 10' strategy

Pupils apply number bonds to 10 to calculate how many more/less to the next multiple of ten. They partition the part into two parts to calculate mentally. Using concrete or pictorial representations can scaffold thinking.

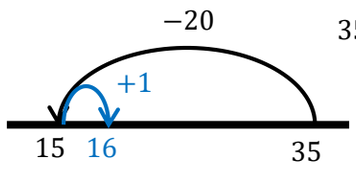
$36 + 27 = ?$ *I can partition 27 into 4 and 23. 36 plus 4 is equal to 40. 40 plus 23 is equal to 63.*





Round and adjust

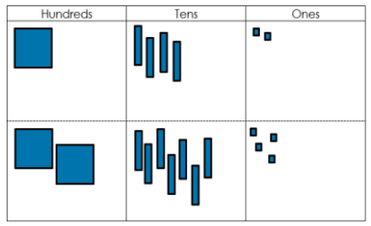
Pupils apply understanding of ordinality of number, recognising when a part or whole is close to a multiple of 10 e.g. 29, 32. They round before calculating, then adjust their answer accordingly. Concrete or pictorial models are used to represent this.



$35 - 19 = 16$

Place value charts

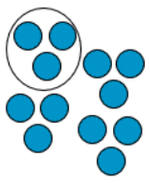
Place value charts are used alongside concrete, pictorial and abstract representations of number to secure understanding of the positional aspect of the number system, particularly when regrouping and exploring multiplication/division by 10 and 100.



Representing fractions

A range of concrete and pictorial representations are used for fractions including fractions of a whole, as part of a set of objects and as part of a quantity such as a length or volume. Pupils should be familiar with a range of representations.

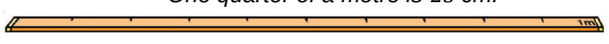
One of four equal parts.



One quarter of 12 *is three.*

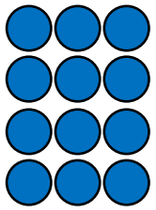
numerator → 1
vinculum →
4 ← *denominator*

One quarter of a metre is 25 *cm.*



Arrays

Concrete and pictorial arrays demonstrate the **commutativity** of multiplication and **inverse relationship** of multiplication and division. Pupils should be familiar with considering rows and columns. **Part-whole language** may be used alongside.

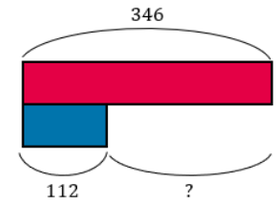


There are four parts/groups each with a value of three. The whole is 12. *Four multiplied by three is equal to* 12.

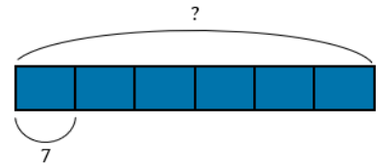
The whole is 12. *There are three parts/groups each with a value of* 4.
12 *divided by three is equal to four. One third of* 12 *is equal to four.*

Bar models and Cuisenaire rods

Pictorial bar models and concrete Cuisenaire as bar models are used throughout the year and represent **part-whole relationships** and **knowns and unknowns** within problems. See PD videos for further exemplification.



I know the whole is 346, *and one of the parts is* 112. *I do not know the value of the missing part. I can subtract* 112 *from* 346.

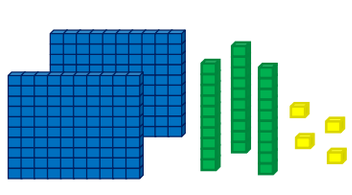


The value of each part is 7 *and there are* 6 *equal parts. The whole is unknown.*

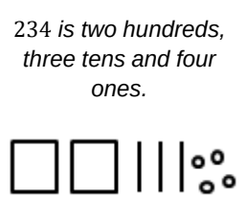
Year 3
Key Methods and Representations

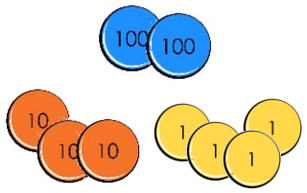
Representations of number

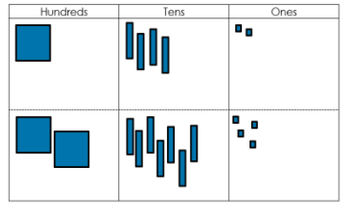
Pupils are familiar with a range of concrete and pictorial representations of number with and without a place value chart. These are used to represent a number or calculation and should not be used as a counting tool. Pupils also make use of these when comparing numbers.



234 is two hundreds, three tens and four ones.







Number lines

Number lines are used to represent and compare, demonstrating the continuous nature of the number system. Wh
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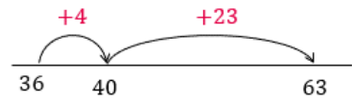
They are also used as a representation for rounding.



Equations

The phrase **'is equal to'** is used consistently to refer to the = symbol. Equations should be presented with symbols and missing numbers in different positions:

$$38 = 25 + 13$$
$$\square = 37 + 44$$
$$12 \div \square = 4$$

Number fact knowledge

Pupils know number bonds to 100 and apply to other multiples of 10. Pupils are increasingly fluent in a range of number facts including partitioning in different ways to discuss number.

136 is multiple of 4 because I can see 120 and 16 which are both multiples of 4.

They are also familiar with multiplication tables for 2. 3. 4. 5. 6, 8 and 10 and related division facts.

$$6 \times 8 = 48 \quad 48 \div 8 = 6$$

Make use of transitions and Maths Meetings to develop this.

Deriving facts and inverse relationships

Pupils use known facts such as number bonds and understanding of place value and magnitude to derive further facts.

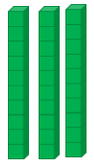
If I know $12 + 5 = 17$ *then* $222 + 5 = 227$
If I know $3 \times 4 = 12$ *then I know* $6 \times 4 = 24$

Inverse relationships have also been explored.

If I know $12 + 5 = 17$ *then* $17 - 12 = 5$
If I know $3 \times 4 = 12$ *then I know* $12 \div 4 = 3$

Multiplication and division by powers of 10

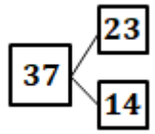
Pupils experience the concept of ten times greater and smaller through exchanging Dienes, linking this to the apparent move of digits in a place value chart.



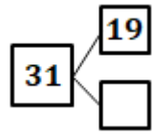
30 is ten times greater than 3.

Part-whole language and representations

A part-whole model is used to represent the relationship between numbers in all four operations. The model is made of a **whole** and two or more **parts**.



The whole is 37.. One part is 23 and one part is 14.



We know the whole is 37 and the value of one part is 19. $37 - 19$ *will give the missing part.*

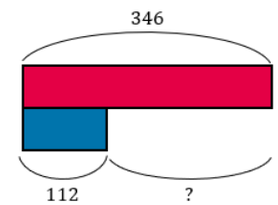
Using multiple **equal** parts represents multiplication, division and fractions of quantities.

16

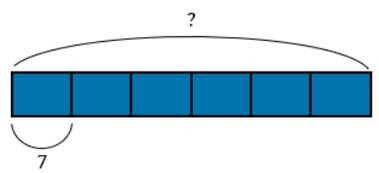
There are three equal parts with a value of four. The whole is 12*.*
Three multiplied by four is equal to 12*.*
12 *divided into three equal parts is equal to four.*
One third of 12 *is four.*

Bar models

Pictorial bar models and concrete Cuisenaire as bar models are used to represent **part-whole relationships** and **knowns and unknowns** within problems in all four operations.



I know the whole is 346*, and one of the parts is* 112*. I do not know the value of the missing part. I can subtract* 112 *from* 346*.*



The value of each part is seven and there are six equal parts. The whole is unknown. Six groups of seven is equal to 42. The whole is 42.

Mental strategies

Pupils experience a range of mental strategies for all four operations, including:

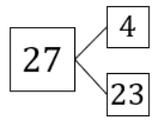
Applying number bonds to 10 and 100 to calculate how many more/less to the next multiple of ten, extending to 100 and 1000 using the 'make 10' strategy.

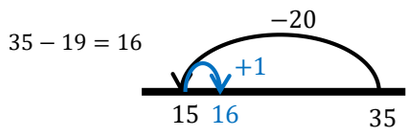
Identifying numbers close to a multiple of ten or 100 e.g. 28, 201 and using a round and adjust strategy, including for multiplication. *"If I know 20 x 4 is 80, then 19 x 4 is 76".*

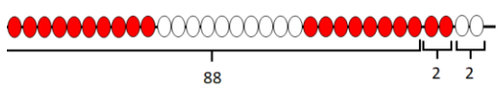
Identifying near doubles for addition. *43 and 45 can be seen as 'double 43 plus two.*

Subtracting numbers close together in value, through counting on to find the difference.

$36 + 27 = ?$ *I can partition 27 into 4 and 23.*
36 plus 4 is equal to 40. 40 plus 23 is equal to 63.





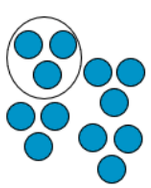


$94 - 88 = ?$ *I can count on from 88 to 94. The difference is 4.*

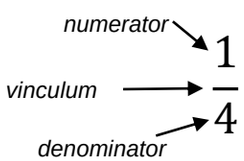
Representing fractions

A range of concrete and pictorial representations are used for fractions including fractions of a whole, as part of a set of objects and as part of a quantity such as a length or volume. Pupils should be familiar with a range of representations and use these to find fractions of a set of quantity.

One of four equal parts.

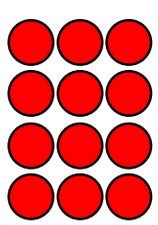


One quarter of 12 *is three.*



Representing multiplicative relationships

Pupils represent multiplicative relationships concretely and pictorially, primarily through arrays, Cuisenaire and bar models. A focus on equal parts, the number of equal parts and the value of each part supports understanding of commutativity and inverse relationships. The representations and language structures support written strategies.



There are four groups each with a value of 3.
There are three groups each with a value of 4.
I can see three, four times.
I can see four, three times.

12 divided into groups of 4 gives three groups
12 shared into four groups gives 3 in each group

Year 4
Key Methods and Representations

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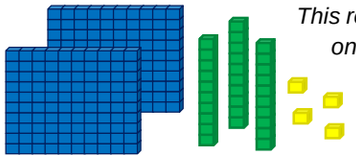
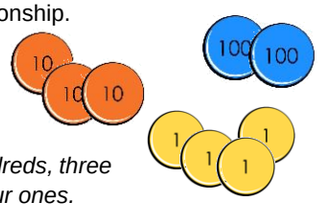
Representations of number

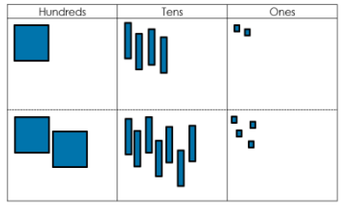
Pupils are familiar with a range of concrete and pictorial representations of number with and without a place value chart. These are used to represent a number or calculation and should not be used as a counting tool. Pupils have also experienced representing decimal numbers using manipulatives including repurposing Dienes equipment, understanding the base 10 relationship.



This represents 2.34. It is 2 ones, 3 tenths and 4 hundredths.

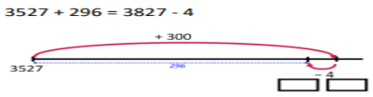


234 is two hundreds, three tens and four ones.



Number lines

Number lines are used to represent and compare, demonstrating the continuous nature of the number system. When calculating, number lines may act as a jotting of the steps of a mental calculation and may begin 'empty' i.e. not have numbered divisions. They are also used as a representation for rounding.



Number fact knowledge

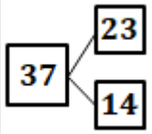
Pupils have an increasing range of number facts. Pupils should know all multiplication tables and related division facts.

Pupils make increasing use of number facts when considering larger integers.

I know 132 is a multiple of 4 because I can partition it into 120 and 12. These are both multiples of 4.

Part-whole language and representations

A part-whole model is used to represent the relationship between numbers in all four operations. The model is made of a **whole** and two or more **parts**.



The whole is 37.. One part is 23 and one part is 14.

Using multiple **equal** parts represents multiplication, division and fractions of quantities.

12

There are three equal parts with a value of four. The whole is 12*. Three multiplied by four is equal to* 12*.*
12 *divided into three equal parts is equal to four.*
One third of 12 *is four.*

Equations

The phrase **'is equal to'** is used consistently to refer to the = symbol. Equations should be presented with symbols and missing numbers in different positions:

38=25+ 13
□=37+44
12 ÷ □ =4

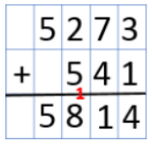
Deriving facts

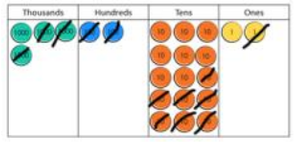
Using known number bonds pupils derive more complex facts including deriving decimal bonds and facts.

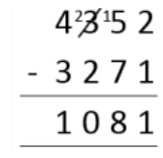
I know $1 + 3 = 4$ *so* $0.1 + 0.3 = 0.4$
I know $13 + 12 = 25$ *so* $1300 + 1200 = 2500$

Written strategies

Pupils are familiar with columnar addition and subtraction, short multiplication and short division written strategies. Manipulatives are used to develop a conceptual understanding. These strategies can be applied to larger integers and decimals.

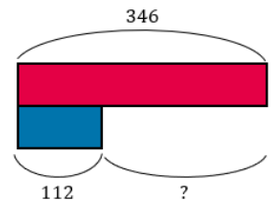




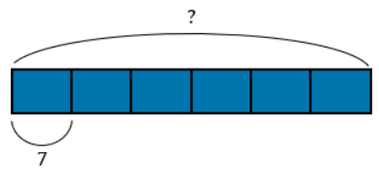


Bar models

Pictorial bar models and concrete Cuisenaire as bar models are used to represent **part-whole relationships** and **knowns and unknowns** within problems in all four operations. See PD videos for further exemplification.



I know the whole is 346*, and one of the parts is* 112*. I do not know the value of the missing part. I can subtract* 112 *from* 346*.*



The value of each part is seven and there are six equal parts. The whole is unknown. Six groups of seven is equal to 42. The whole is 42.

Mental strategies

Pupils experience a range of mental strategies for all four operations, including:

Applying number bonds to 10 and 100 to calculate how many more/less to the next multiple of ten, extending to 100 and 1000 using the 'make 10' strategy.

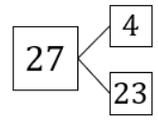
Identifying numbers close to a multiple of ten or 100 e.g. 28, 201 and using a round and adjust strategy, including for multiplication. *"If I know 20 x 4 is 80, then 19 x 4 is 76".*

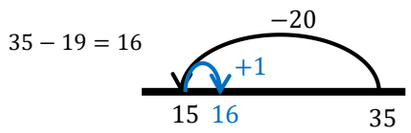
Identifying near doubles for addition. *43 and 45 can be seen as 'double 43 plus two.*

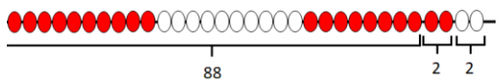
Subtracting numbers close together in value, through counting on to find the difference.

Once secure, these can be applied to larger integers and decimal values.

$36 + 27 = ?$ *I can partition 27 into 4 and 23.*
36 plus 4 is equal to 40. 40 plus 23 is equal to 63.





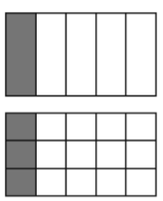


$104 - 98 = ?$ *I can count on from 98 to 104.*
The difference is 6.

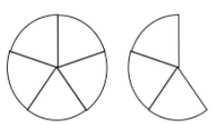
Representing fractions

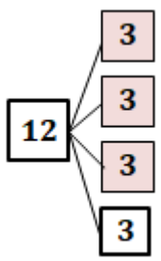
Pupils will represent unit, non-unit and improper fractions in a variety of ways including area, part of a set and on a number line. Through representations they understand equivalence.

$\frac{1}{4} = \frac{3}{12}$

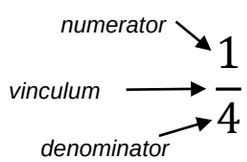


$\frac{8}{5} = 1\frac{3}{5}$



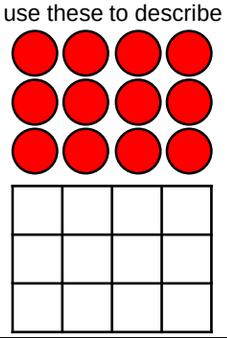


$\frac{3}{4}$ *of 12 is equal to 9.*



Representing multiplicative relationships

Pupils use an increasing range of models to represent multiplicative relationships and use these to describe inverse relationships and commutativity.



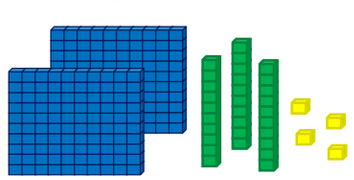
There are three rows with a value of four. There are four columns with a value of 3.
$3 \times 4 = 12$
$4 \times 3 = 12$
$12 \div 4 = 3$
$12 \div 3 = 4$

Three groups of four are equal to 12.
Four groups of three are equal to 12.

Year 5
Key Methods and Representations

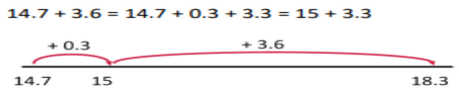
Representations of number

Pupils are familiar with a range of concrete and pictorial representations of number with and without a place value chart. These are used to represent a number or calculation and should not be used as a counting tool. Pupils experience representing decimal numbers using manipulatives including repurposing Dienes equipment, understanding the base 10 relationship.



Number lines

Number lines can be used to represent and compare, demonstrating the continuous nature of the number system. When calculating, number lines may act as a jotting of the steps of a mental calculation and may begin 'empty' i.e. not have numbered divisions. They are also used as a representation for rounding.



Number fact knowledge

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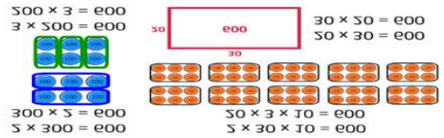
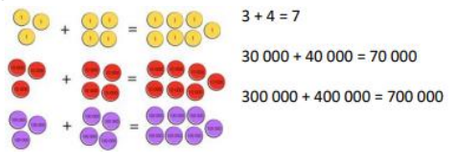


Deriving facts

Using the following language makes the logic explicit:
I know three ones plus four ones is equal to seven ones.
Therefore, three ten thousands plus four ten thousands is equal to seven ten thousands.



Written strategies

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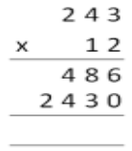

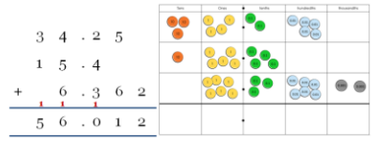

Part-whole language and representations

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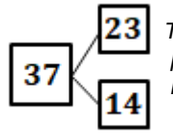
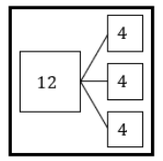


The whole is 37.. One part is 23 and one part is 14.

Using multiple equal parts represents multiplication, division and fractions of quantities.

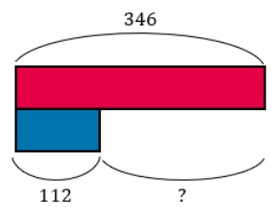


There are three equal parts with a value of four. The whole is 12. Three multiplied by four is equal to 12.
12 divided into three equal parts is equal to four.
One third of 12 is four.

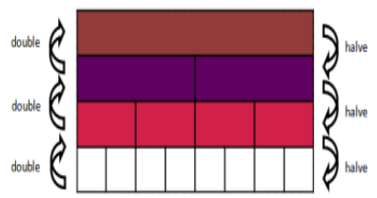
Close links are made between this and bar model representations.

Bar models

Pictorial bar models and concrete Cuisenaire as bar models are used to represent **part-whole relationships** and **knowns and unknowns** within problems in all four operations.



I know the whole is 346, and one of the parts is 112. I do not know the value of the missing part. I can subtract 112 from 346.



Multiply by 8 by doubling three times

e.g. 12 × 8 = 24 × 4 = 48 × 2 = 96

Divide by 8 by halving three times

e.g. 104 ÷ 8 = 52 ÷ 4 = 26 ÷ 2 = 13

Fluency and Calculation strategies

Pupils experience a range of fluency and calculation strategies for all four operations, including:

Dividing whole numbers and those involving decimals by 10, 100 or 1000.

Multiplying whole numbers and those involving decimals by 10, 100 or 1000.

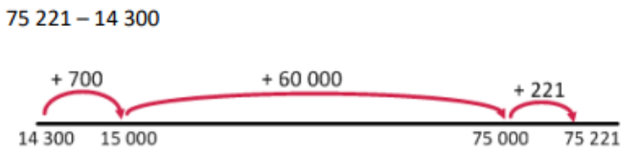
Converting units of measure.

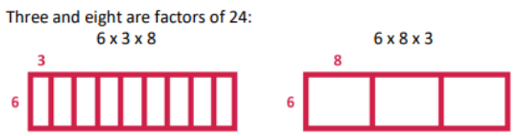
Recalling percentage and decimal equivalents of 1/ 2, 1/ 4, 1/ 5, 2/ 5, 4/ 5 and fractions with a denominator of a multiple of 10 or25.

Adding and subtracting whole numbers with more than 4 digits.

Identifying multiples and factors, including all factor pairs of a number, and common factors of 2 numbers.

102.14 x 10 = 1021.4

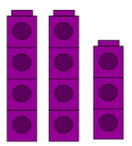




210.3 ÷ 10 = 21.03

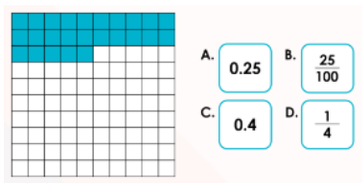
Representing fractions

Pupils will represent unit, non-unit and improper fractions in a variety of ways including area, part of a set and on a number line. They will now extend their knowledge of fractions to thousandths, and will learn about percentages.



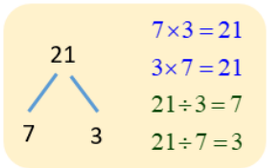
$2\frac{3}{4} = \frac{11}{4}$

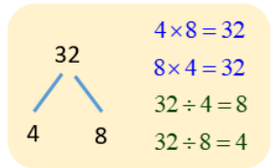
$\frac{70}{100} = 70\%$



Representing multiplicative relationships

Pupils use an increasing range of models to represent multiplicative relationships and use these to describe inverse relationships and commutativity.





Three groups of four are equal to 12.
Four groups of three are equal to 12.